Field Displacement Phenomenon in a Rectangular Waveguide Containing a Thin Plate of Ω Medium

Jerzy Mazur and Dariusz Pietrzak

***Abstract*—An analysis of the wave propagation in a rectangular waveguide containing pseudochiral Ω slab is presented in this letter. Approximate continuity conditions modeling a thin Ω plate are introduced to simplify the analysis. The problem is solved by the mode-matching procedure. A notable field displacement phenomenon appearing in the guide owing the pseudochirality properties is studied. Other feature such as a weak perturbation of the dispersion characteristics of the isotropic guide by pseudochirality introduced into the medium is also discussed.**

I. Introduction

ELECTROMAGNETIC properties of the new complex materials, named pseudochiral or Ω-media, have recently generated considerable attention in the literature [1]–[3]. Although the Ω-media are nonchiral, their properties are governed by constitutive relations similar to the ones describing chiral media. The chiral material consists of small wire helixes inserted into the host medium while the typical pseudochiral medium contains Ω-shaped microstructures in which both the loop and stamps lie in the same plane. From this arises the different interaction of electric and magnetic fields of high frequency in both of these materials. Although the electric field in both the wire and the Ω elements induces not only electric but also magnetic polarization and vice versa, differences in the mutual placement of the polarization vectors are observed. In the wire element of the chiral medium, these interacting fields are parallel while in the Ω element of the pseudochiral medium they are perpendicular to each other. In particular, the chiral medium exhibits optical activity that refers to the rotation of the polarization plane [7]. This effect does not occur in the Ω medium where the field displacement phenomenon takes place [5], [6]. Except for nonreciprocity, these effects are similar to the ones appearing in gyrotropic materials (ferrite, plasma).

It seems that the pseudochiral media have more features that make them prospective for applications in novel integrated microwave and millimeter wave devices. Such media can be easily made by spreading conducting Ω-shaped microstructures as homogeneous matrix in or on the host medium and their orientation can be established freely. Note here that the different field features of the electromagnetic wave in such media occur when the orientation of the Ω elements in the material is changed. Furthermore the Ω element matrix can also be printed on the different kinds of substrates, such as dielectric, ferrite or semiconductor, which suggests more possibile applications. In [4] electromagnetic properties of the rectangular waveguide containing Ω plate were presented and a reciprocal phase shifter design was suggested. The electromagnetic wave interaction in ferrite-pseudochiral materials was treated in [5] and [6]. The nonreciprocal field displacement phenomena observed in [6] was utilized to propose a new type four-port circulator based on the section of coupled line filled with Ω-ferrite medium.

In this letter, we propose a mathematical explanation of the field displacement effect appearing in rectangular waveguide containing thin plate of pseudochiral medium. A thin Ω plate is placed in the y-z plane. The stamps of the Ω elements are laid along the z-axis parallel to the wave propagation direction and the loops orientation is the positive direction of the y axis. To simplify the analysis of the problem, we derive approximate continuity conditions that can be used to model the thin pseudochiral slab. Next, the analytical model of electromagnetic wave propagation in the considered guide is formulated using mode-matching technique. The main feature of the line is that the field displacement along y axis is observed and the magnitude of the effect depends on the Ω plate parameters.

II. Theory

Let us consider a two-dimensional (2–D) slab of Ω medium with thickness d as shown in Fig. 1. whose constitutive relations are of the form

$$\begin{aligned} \vec{D} &= \varepsilon_0 \bar{\bar{\varepsilon}} \vec{E} + j\bar{\bar{\Omega}}_{zx}\vec{B} \\ \vec{B} &= \mu_0\bar{\bar{\mu}}_c\vec{H} - j\mu_0\bar{\bar{\mu}}_c\bar{\bar{\Omega}}_{xz}\vec{E} \end{aligned} \tag{1}$$

where the relative electric permittivity and magnetic permeability of the medium are defined via diagonal dyadics

$$\begin{aligned} \bar{\bar{\varepsilon}} &= \varepsilon_{xx}\vec{i}_x\vec{i}_x + \varepsilon_{yy}\vec{i}_y\vec{i}_y + \varepsilon_{zz}\vec{i}_z\vec{i}_z \\ \bar{\bar{\mu}}_c &= \mu_{xx}\vec{i}_x\vec{i}_x + \mu_{yy}\vec{i}_y\vec{i}_y + \mu_{zz}\vec{i}_z\vec{i}_z\,. \end{aligned} \tag{2}$$

For the considered pseudochiral slab the coupling dyadics take the form

$$\bar{\bar{\Omega}}_{xz} = \Omega_c\vec{i}_z\vec{i}_x \quad \bar{\bar{\Omega}}_{zx} = \Omega_c\vec{i}_x\vec{i}_z. \tag{3}$$

The parameter Ω_c is pseudochiral admittance and represents the coupling between electric and magnetic field along z and x axes, respectively. In general we can infer from the

Manuscript received July 27, 1995. This work was supported by the Polish Committee for Scientific Research under Contract PB0745/P4/93.

The authors are with the Faculty of Electronics, Technical University of Gdańsk, Poland.

Publisher Item Identifier S 1051-8207(96)00451-5.

1051–8207/96$05.00 © 1996 IEEE

Maxwell equations that in an unbounded pseudochiral medium the hybrid modes propagate. For a medium described by the constitutive equation (1) we insert (1) into the Maxwell equations and eliminate field components normal to the surface of the slab. What we get is the following set of coupled partial differential equations that describes behavior of the tangential electric and magnetic field components:

$$\begin{aligned}\nabla_x E_z &= -j\frac{\beta_z}{k_0\varepsilon_x}\nabla_y \eta H_z + \frac{\beta_z^2 - k_0^2\mu_y\varepsilon_x}{k_0\varepsilon_x}\eta H_y\\ \nabla_x E_y &= \frac{\nabla_y^2 + k_0^2\mu_z\varepsilon_x}{k_0\varepsilon_x}\eta H_z + j\frac{\beta_z}{k_0\varepsilon_x}\nabla_y\eta H_y\\ \nabla_x \eta H_z &= -j\frac{\beta_z}{k_0\mu_x}(\nabla_y + k_0\mu_x\eta_0\Omega_c)E_z + \frac{\beta_z^2 - k_0^2\mu_x\varepsilon_y}{k_0\mu_x}E_y\\ \nabla_x \eta H_y &= \frac{\nabla_y^2 + k_0^2\mu_x\varepsilon_z}{k_0\mu_x}E_z + j\frac{\beta_z}{k_0\mu_x}(\nabla_y - k_0\mu_x\eta_0\Omega_c)E_y\end{aligned} \tag{4}$$

where an $\exp(-j\beta z)$ dependence is assumed and $\eta = -j\eta_0$. Approximate continuity conditions that relate the tangential field components $\{F\} = \{E_z, E_y, \eta H_z, \eta H_y\}$ on both interfaces x_1 and x_2 of the thin slab follow from (4) if the following assumptions concerning both transition of the tangential field components across the slab and an average field components inside the medium are made:

$$\nabla_x F = \frac{F\big|_{x_1} - F\big|_{x_2}}{d} \quad \text{and} \quad F = \frac{F\big|_{x_1} + F\big|_{x_2}}{2}. \tag{5}$$

Note here that the factors in (4) dependent on parameter Ω_c evoke the coupling between orthogonal modes of the dielectric guide. It induces that the field displacement effect can be expected in the investigated structure. Since it will be difficult to obtain a simple analytical solution, a mode-matching approach is used to solve a stated problem. First, the tangential to the slab longitudinal electric and magnetic field components in the empty regions 1 and 2 of the guide are expanded into Fourier series taking the boundary conditions on the screening wall

$$\begin{aligned}E_z^{(1,2)} &= \sum_{n=1}^{\infty} A_n^{(1,2)(e)}\sinh(\alpha_n^{(1,2)}x_i)\sin(k_n y)\\ \eta H_z^{(1,2)} &= \sum_{n=0}^{\infty} A_n^{(1,2)(h)}\cosh(\alpha_n^{(1,2)}x_i)\cos(k_n y)\end{aligned} \tag{6}$$

where $x_i = x$ in region 1 and $x_i = (a - x)$ in region 2, $k_n = n\pi/b$ and

$$\alpha_n^{(1,2)} = \sqrt{k_n^2 + \beta_z^2 - k_0^2\mu_r^{(1,2)}\varepsilon_r^{(1,2)}}. \tag{7}$$

The remaining tangential field components E_y and ηH_y are derived from relations between transverse and longitudinal components. Inserting the tangential field components defined on the both interfaces of the Ω slab into (4) we bring our problem to four homogeneous functional equations. Next the moment procedure is applied to solve the equations. The expansion harmonic functions are taken into account as weight functions. As a result of the procedure we obtain an infinite set of homogeneous equations that can be expressed in the matrix form as follows:

$$\underline{\underline{Z}}(\beta_z)[\bar{A}] = 0 \tag{8}$$

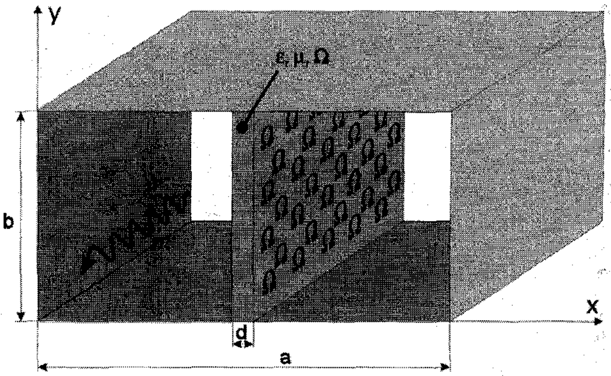

Fig. 1. A structure under examination—A rectangular waveguide containing a longitudinal pseudochiral Ω-slab.

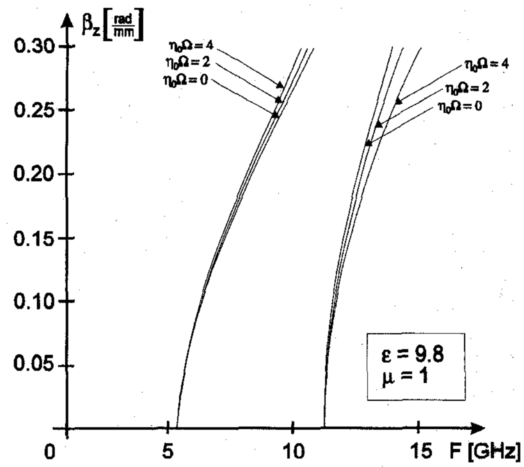

Fig. 2. Dispersion characteristics calculated for the two first modes propagating in the structure under examination.

where

$$[\bar{A}] = [A_0^{1h}, A_0^{2h}, A_1^{1e}, \ldots].$$

The determinant of matrix $Z(\beta_z)$ is set to zero, yielding the dispersion equation for hybrid modes in the guide.

III. Numerical Results

Using the described mathematical model, a rectangular waveguide structure of size 20 × 10 mm was examined. The pseudochiral longitudinal Ω slab of thickness $d = 1$ mm was placed symmetrically inside the waveguide as shown in Fig. 1. For the structure the dispersion characteristics of the two first modes were calculated as shown in Fig. 2. Note that although the normalized coupling coefficient $\eta_0\Omega$ is changing strongly, the dispersion curves deviate very slightly. In spite of this, the variation of the coupling coefficient strongly affects the field distribution inside the waveguide. Fig. 3 shows the power density distribution calculated for an empty waveguide and, for comparison, for a waveguide filled with a slab of dielectric and pseudochiral material. Note that the field displacement phenomenon can be observed when the pseudochirality is introduced. Intensity of the phenomenon depends both on the coupling coefficient and the frequency of the signal. As an example we calculated the power density distribution along the interface of the slab ($x = x_1$) for different values of $\eta_0\Omega$

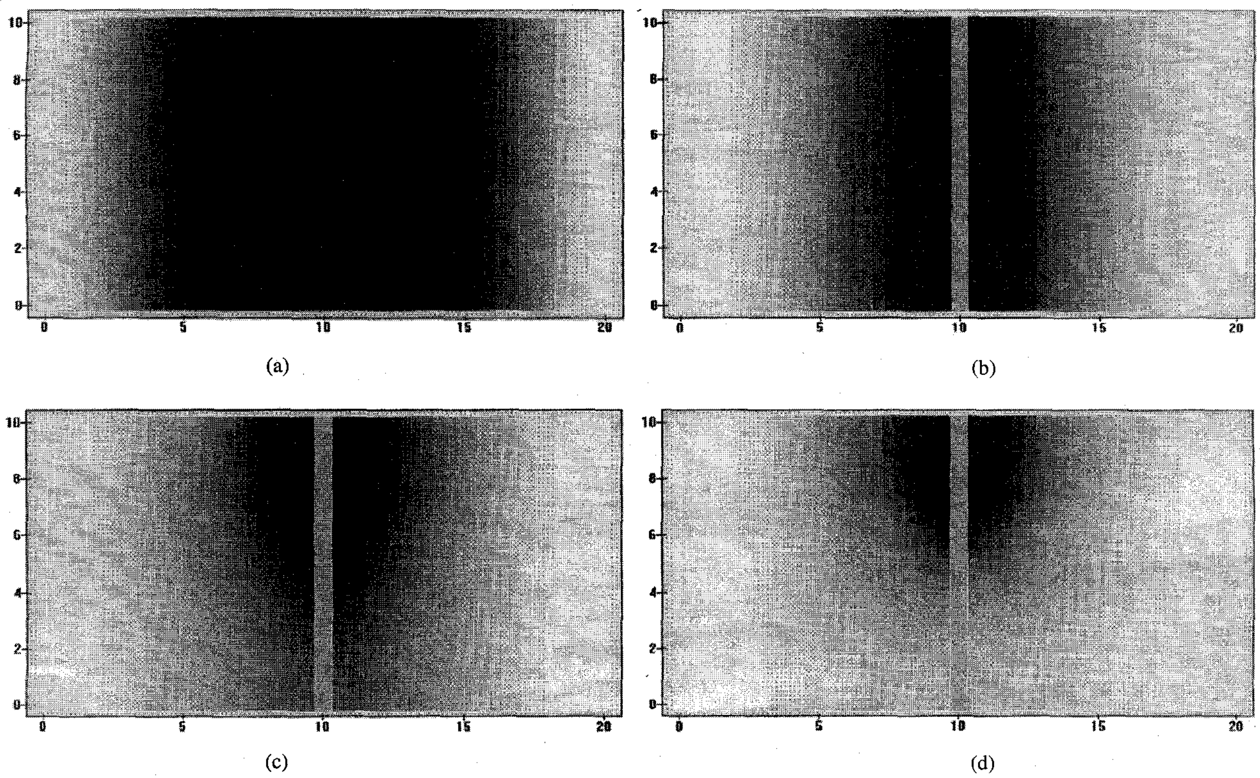

Fig. 3. A power density distribution calculated for an empty waveguide (a) and for the same waveguide containing a dielectric slab of thickness $d = 1$ mm (b). Note the field distribution phenomenon in the case when pseudochirality is introduced: $\eta_0 \Omega = 2$ (c) and $\eta_0 \Omega = 4$ (d). $F = 9.5$ GHz.

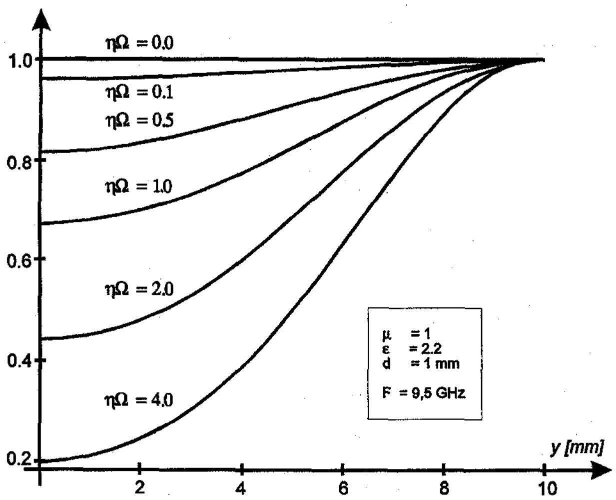

Fig. 4. A power density distribution at the Ω slab interface calculated for different values of the normalized coupling coefficient $\eta_0 \Omega$. The curves are normalized to the maximum point value.

as it is shown in Fig. 4. The intensity of the displacement is proportional to the change in $\eta_0 \Omega$.

The method used for calculation gives good accuracy as long as the slab is thin enough. It has been examined that for the dielectric slab, i.e. for the only case when the strict solution of the dispersion equation can be found, the relative error is not greater than 1% for d/λ less than 0.03 and not greater than 5% for d/λ less than 0.10.

In summary, a rectangular waveguide with a thin slab of pseudochiral Ω medium has been analyzed by the mode-matching technique. Approximate continuity conditions have been introduced to model the Ω plate and to simplify the analysis. Several unique and notable features associated with the structure under consideration have been presented. It has been shown that this structure support hybrid modes only. Although a weak perturbation of the dispersion characteristics of the modes in an isotropic guide is observed as the pseudochirality is inserted into the pseudochiral medium, it has a strong effect upon the field distribution inside the structure. It has been shown that in such a structure a field displacement phenomenon can be observed.

REFERENCES

[1] N. Engheta and M. Saadoun "Novel pseudochiral or omega medium and its application," in *Proc. Conf. PIERS '91*, Cambridge, MA, July 1991, vol. 1, pp. 01–05.

[2] M. Saadoun and N. Engheta "The pseudochiral omega medium. What is it and what can it be used for?," in *Dig. Conf. URSI, Conf. IEEE AP-S Int. Symp.*, Chicago, IL, 1992, vol. 4, pp. 2038–2043.

[3] ——, "Pseudochiral omega medium and guided wave structures: Theory and principles," in *Conf. URSI Int. Symp. Electromagnetic Theory*, Sydney, Aug. 17–20, 1992, pp. 17–20.

[4] ——, "A reciprocal phase shifter using novel pseudochiral or omega medium," *Microwave Optic. Technol. Lett.*, vol. 5, no. 4, pp. 184–186, 1992.

[5] S. Tretyakov, "Thin pseudochiral layers: Approximate boundary conditions and potential applications," *Microwave Optic. Technol. Lett.*, vol. 6, no. 2, pp. 112–115, 1993.

[6] J. Mazur, "Nonreciprocal phenomena in coupled lines containing ferrite-pseudochiral omega media," in *40 Int. Wissenschaft. Colloquium*, Ilmenau, Germany, Sept. 18–21, 1995.

[7] P. Pelet and N. Engheta, "The theory of chirowaveguides," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 1, pp. 90–98, Jan. 1990.